Enhancement of the spin gap in fully occupied two-dimensional Landau levels

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Polarization-resolved magnetoluminescence, together with simultaneous magnetotransport measurements, have been performed on a two-dimensional electron gas (2DEG) confined in CdTe quantum well in order to determine the spin splitting of fully occupied electronic Landau levels, as a function of the magnetic field (arbitrary Landau-level filling factors) and temperature. The spin splitting, extracted from the energy separation of the σ^+ and σ^- transitions, is composed of the ordinary Zeeman term and a many-body contribution which is shown to be driven by the spin polarization of the 2DEG. It is argued that both these contributions result in a simple, rigid shift of Landau-level ladders with opposite spins.

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I. INTRODUCTION

A number of experiments on two-dimensional electron gases (2DEGs) (Refs. 1–4) clearly show that the thermal activation of carriers across the Fermi energy, located between the spin-split Landau levels (LLs) at odd integer filling factors (ν), is governed by a gap which can significantly surpass the single-particle Zeeman energy included in band-structure models. This phenomenon, referred to in the literature as g factor or spin-gap enhancement,[5–7] is thought to be driven by the spin polarization of a 2DEG and is a primary manifestation of the interactions between two-dimensional electrons in the integer quantum Hall effect (QHE) regime. It is a result of the specific character of the spin-excitation spectra of a 2DEG at odd integer ν-QHE states.[8,9] It can be seen as arising from the contribution of Coulomb interactions (including exchange terms) to the energy which is required to remove, or inject, an electron from, or to, a given spin-resolved LL.

To date, the effect of the spin-gap enhancement has been generally limited to experiments[1,2,4,10–12] which probe the spin splitting at the Fermi level, for QHE states at exactly odd filling factors. This limitation has been thought to be overcome with spectroscopic methods such as, for example, interband optics[13–15] or tunneling experiments,[16] which, within their trivial description, permit to investigate the processes of removing/adding an electron from/to a 2DEG, at arbitrary energy, filling factor and temperature. Among the different spectroscopic methods, magnetoluminescence measurements has been widely invoked to investigate electron-electron correlation in the QHE regime, however, measurements to probe the spin-gap enhancement are rather scarce.[13,16]

Here, we report on magnetophotoluminescence studies of a 2DEG confined in a high-quality CdTe quantum well (QW), and, show that the enhancement of the spin splitting is not only a property of spin excitations at the Fermi level but that it is also relevant for fully occupied spin Landau levels, located well below the Fermi energy. We have measured the many-body contribution to the spin gap for fully populated spin Landau levels over a wide range of filling factors and temperatures, and show that it is driven by Coulomb interaction, apparent via the spin polarization of the investigated 2DEG with its relatively large bare Zeeman splitting.

The increasingly high quality of GaAs/GaAlAs structures has been driving advances in the physics of interacting 2D electrons. Notably, 2D electrons in a GaAs matrix are characterized by a relatively small bare g factor (−0.44) and therefore by a small value of the interaction parameter $\eta = E_z/\mathcal{D}$, where $E_z = g\mu_B B$, $\mathcal{D} = e^2/\epsilon l_B$, and, $l_B = \sqrt{\hbar/eB}$ is the magnetic length. The small value of η is responsible for the rich physics exhibited by interacting 2D electrons in the QHE regime, for example, the occurrence of competing spin-polarized/spin-unpolarized many-body ground states[17] or Skyrmion-type spin-texture excitations.[18–20] However, this complex physics often masks the appearance of simpler and basic many-body effects, which should emerge more clearly when η is sufficiently large. Disorder is an additional source of complications in ascertaining the spin polarization in systems with small g factors. While high electron mobilities are obviously advantageous, GaAs-based structures are also rather fragile, displaying, for example, metastable effects upon illumination, with an associated decrease in mobility and homogeneity, which frequently prevents the simultaneous basic characterization of such structures using magneto-optics and magnetotransport. A 2DEG in a CdTe matrix,[21] used in our experiments, is characterized by relatively large (bare) g factor (−1.6) and the η parameter in this system exceeds by a factor of ≈ 3 its value in GaAs structures (the dielectric screening $\epsilon = 10$ is slightly less efficient in CdTe). CdTe, which has a conduction band as simple as the one in GaAs, appears to be an almost ideal model system to study the QHE physics of the primary spin-polarized states. The significant progress in the crystal growth of CdTe quantum wells permits nowadays to attain a 2DEG with reasonably high mobilities. As shown in Fig. 1, the sample studied here shows a well-pronounced fractional QHE and permits a trouble-free, simultaneous measurement of high-quality magnetophotoluminescence and magnetotransport.

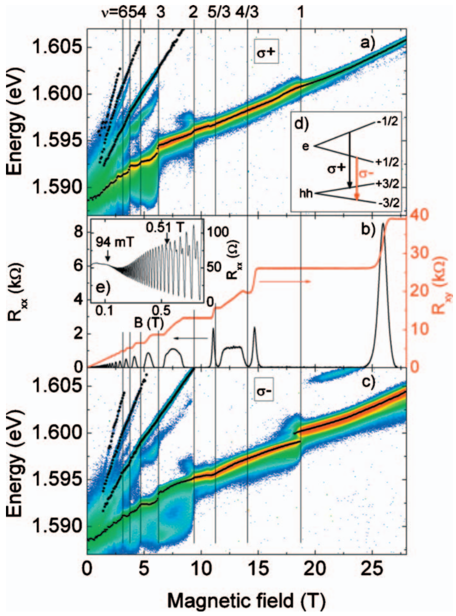

FIG. 1. (Color) (a) and (c) Color plot of σ^+ and σ^- magneto-PL of a 2DEG in a CdTe QW, measured at 80 mK, under low power (≈ 0.5 W/m^2), $\lambda = 514$ nm Ar^+ excitation. Black points indicate the energy of the main peaks. Inset (d) shows the optical selection rules. (b) Results of simultaneous magnetotransport measurements showing the longitudinal (R_{xx}) and Hall (R_{xy}) resistance. Vertical lines indicate the Landau-level filling factor. Inset (e) shows an expanded view of R_{xx} at low magnetic fields.

II. SAMPLES

The active part of the investigated structure consists of a 20-nm-wide CdTe QW, modulation doped on one side with iodine, and embedded between $Cd_{0.74}Mg_{0.26}Te$ barriers. The sample, in form of 1.5×6 mm^2 rectangle, was equipped with electrical contacts in a Hall-bar configuration to permit simultaneous optical and electric measurements. Experiments have been carried out using either a ^{3}He/^{4}He dilution refrigerator or a variable temperature ^{4}He cryostat, in magnetic fields supplied by a resistive (28 T) or superconducting (11 T) magnets. A standard, low-frequency (≈ 10 Hz) lock-in technique has been applied for the resistance measurements. Polarization-resolved, σ^+ and σ^- photoluminescence (PL) spectra have been measured using a single 600-μm-diameter optical fiber to transmit the excitation beam (514 nm line of Ar^+ laser) and to collect the photoluminescence signal for the spectrometer (spectral resolution ≈ 100 μeV) equipped with a charge coupled device camera. An appropriate linear polarizer and $\lambda/4$ plate were placed directly between the end of the fiber and the sample. The σ^+ and σ^- PL components were measured by reversing the polarity of the magnetic field. Special attention has been paid to assure a low level of laser excitation (≈ 50 μW/cm^2), to precisely calibrate the magnetic field, and to measure the spectra at small intervals (down to 5 mT) of the magnetic field. Under our experimental conditions (continuous laser illumination), the 2DEG density of $\approx 4.5 \times 10^{11}$ cm^{-2} and mobility of $\mu = 2.6 \times 10^5$ cm^2/V s were well reproduced in different experimental runs.

III. EXPERIMENTAL RESULTS

The representative results of simultaneous magneto-PL and magnetoresistivity measurements of our sample are shown in Fig. 1. As can be seen in Fig. 1(b), the investigated 2DEG shows all typical attributes of the QHE in a system with fairly high mobility and relatively high electron concentration; well-developed integer QHE states and the appearance of 5/3, 4/3, and 2/3 fractional states (which will be discussed elsewhere)[22]. From the field at which the Shubnikov de Haas (SdH) oscillations ($B_1 \approx 94$ mT), and spin splitting appears ($B_2 \approx 0.51$ T), we obtain a first estimate of the enhanced g factor, $g^* \approx 3.7$ using the condition ($\hbar e B_1 / m^* \approx g^* \mu_B B_2$) where the electron effective mass $m^* = 0.1 m_e$ was derived from cyclotron resonance absorption measured on a parent sample. A Dingle analysis of the SdH oscillations gives a quantum lifetime $\tau_q = \hbar / 2\Gamma = (3.0 \pm 0.3)$ ps (broadening of Lorentzian Landau levels $\Gamma \approx 110$ μeV) as compared to the transport lifetime $\tau_\tau \approx 15$ ps (derived from the measured mobility).

The evolution of the PL with the magnetic field (Fig. 1) resembles spectra reported in numerous PL investigations, widely applied in the past to GaAs-based structures.[23–29] Peaks in the magneto-PL spectra are due to the recombination of electrons from occupied conduction-band LLs $[L_N, E_N = (N+1/2)\hbar\omega_c, N = 0, 1, \ldots]$ with photoexcited holes from valence-band LLs $[L_N^h, E_N^h = (N+1/2)\hbar\omega_c^h, N = 0, 1, \ldots]$, where ω_c and ω_c^h are the cyclotron frequency of the electrons and holes, respectively. The energy of the main peaks, due to $L_N \rightarrow L_N^h$ $(N_e - N_h = 0)$ transitions which scale as $E_0 + (N+1/2)(\hbar\omega_c + \hbar\omega_c^h)$, are shown as black dots in Fig. 1. Since $\hbar\omega_c / \hbar\omega_c^h \approx 5$,[30] the magneto-PL spectra reflect largely the characteristic fan chart of electronic LLs (with respect to band-edge energy, E_0) including their occupation factor. Opposite LL spin components are resolved in the σ^+ and σ^- spectra. The exchange of the intensity between the σ^+ and σ^- PL when sweeping through filling factor $\nu = 1$ is typical of the 2DEG studied here and results from the selection rules which are specific to CdTe [see Fig. 1(d)].

The nonmonotonic variation, with magnetic field, of the transition energies and intensities (oscillations which correlate with filling factor) and possible appearance of line splitting (see Fig. 1) are other common features of magneto-PL investigations of a 2DEG. Electron-electron interactions, combined with different perturbations induced by the presence of the valence-band hole, are almost certainly at the origin of these features.[23–29] The understanding these features is far from universal and a detailed analysis of the energy and intensity of each individual magneto-PL transitions is beyond the scope of our paper. We have found, however, that information on the effects of electron-electron interactions can be extracted from the relative positions of polarization-resolved PL peaks arising from different LL spin components.

IV. SPIN SPLITTING OF FULLY POPULATED LANDAU LEVELS

We focus our attention on the two lowest energy σ^+ and σ^- magneto-PL transitions (Fig. 2) which are due to elec-

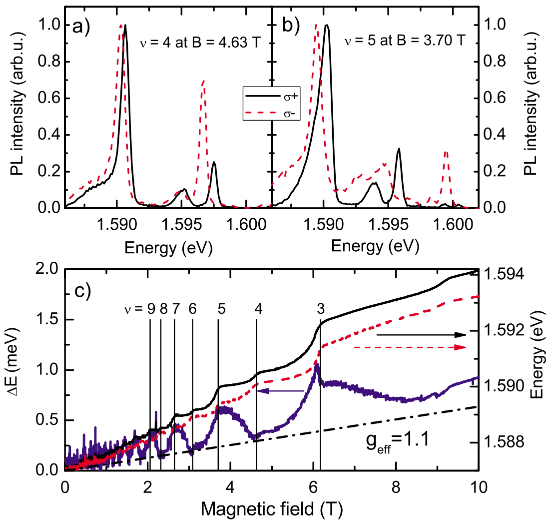

FIG. 2. (Color online) Normalized σ^+ and σ^- PL spectra at magnetic fields corresponding to filling factors (a) $\nu=4$ and (b) $\nu=5$ ($T=80$ mK, $E_{exc}=2.41$ eV, and $P_{exc}=0.5$ W/m^2). (c) Magnetic field dependence of the energy of polarization resolved transitions from the different spin levels of the $N=0$ electronic LL (right scale) and their splitting (left scale). Dashed-dotted line shows the ordinary Zeeman effect in the conduction and valence bands.

trons, with different spins, recombining from the fully populated (L_0) LL. While the energy of each of these peaks displays a nontrivial dependence on the magnetic field, here we focus on the evolution of the energy separation ΔE between the σ^+ and σ^- transitions plotted in Fig. 2(c). The splitting ΔE does not follow a linear field dependence which is expected for the case of an ordinary Zeeman effect. This can be even seen in the raw data in Figs. 2(a) and 2(b); The splitting ΔE observed at higher field $B=4.63$ T is clearly smaller than the splitting at lower fields $B=3.7$ T. The ΔE versus B dependence in Fig. 2(c) naturally suggests that this dependence is composed of two terms; a Zeeman term (ΔE_Z) linear with B and a many-body term ($\Delta E_{\uparrow\downarrow}$), which is nonmonotonic with B, having maxima at odd integer ν and zeros at even integer ν. The linear term can be extracted from the splitting at even integer ν and it is in agreement with the ordinary Zeeman effect expected in our structure. Taking into account the selection rules depicted in Fig. 1(d), the splitting $\Delta E_Z=(|g|-|g_h|)\mu_B B=g_{eff}\mu_B B$, which requires $g_{eff}=1.1$ to fit the data in agreement with the reported values of $g=-1.6$ and $g_h\approx 0.5$, for electronic and valence hole g factors in CdTe QWs.[31]

V. DATA MODELING

To further clarify the origin of the $\Delta E_{\uparrow\downarrow}$ term, we plot this term as a function of filling factor and show its characteristic evolution with temperature (Fig. 3). We have extracted $\Delta E_{\uparrow\downarrow}$ from different experimental runs, by subtracting the ordinary Zeeman term which is assumed to be temperature independent. The electron concentration (filling factor scale) was determined using simultaneous magnetoresistance measurements. An inspection of the results presented in Fig. 3 strongly suggests that $\Delta E_{\uparrow\downarrow}$ is ruled by the spin polarization $\mathcal{P}=\frac{n_\downarrow - n_\uparrow}{n_\downarrow + n_\uparrow}$ of the 2DEG. A quantitative verification of this hypothesis is provided by the following simple model. We consider the ideal case of a 2DEG with discrete Landau levels separated by $\hbar\omega_c$ and spin split by

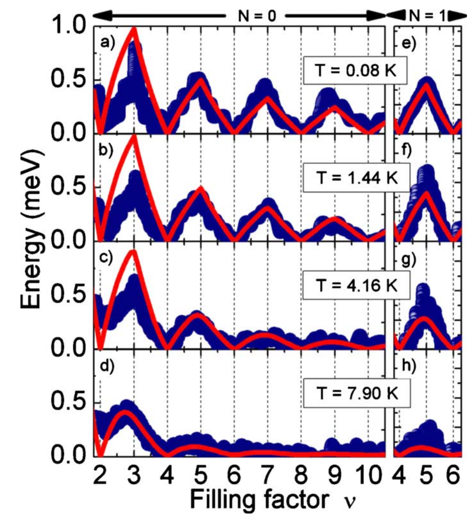

FIG. 3. (Color online) Many-body contribution to the spin gap in the $N=0$ (left panel) and $N=1$ (right panel) Landau level extracted from magneto-PL experiments (points) at different temperatures. Traces calculated using the model described in the text (solid lines) are shown for comparison.

$$\Delta_s = |g|\mu_B B + \Delta E_{\uparrow\downarrow} = |g|\mu_B B + \Delta_0'\varphi(B)\frac{n_\downarrow - n_\uparrow}{n_\downarrow + n_\uparrow}. \qquad (1)$$

In particular, we assume that the enhanced part ($\Delta E_{\uparrow\downarrow}$) of the spin splitting is common for all Landau levels, including the lowest LL (L_0) which we probe with PL and the LL in the vicinity of the Fermi energy, the occupation of which determines the spin polarization. Furthermore, we suppose that $\varphi(B)=\sqrt[4]{B^2+B_0^2}$ in order to phenomenologically account for the expected behavior of $\Delta E_{\uparrow\downarrow}$ in the limit of high magnetic fields [$\varphi(B)\sim\sqrt{B}$] and when B tends to zero [$\varphi(B)=$constant].[32] Finally, we self-consistently calculate $\Delta E_{\uparrow\downarrow}$ (and $\mathcal{P}$) and obtain agreement with the data by adjusting the two fitting parameters, $\Delta_0=\Delta_0'\sqrt{B_0}=2.1$ meV and $B_0=3.7$ T.

VI. DISCUSSION

Despite the rather crude approximations, the calculations well reproduce the experimental data (Fig. 3) over a wide range of filling factors ($4\leq\nu\leq 10$) and for different temperatures up to the temperature for which $\Delta E_{\uparrow\downarrow}$ (and $\mathcal{P}$) vanishes. The agreement is less satisfactory in the vicinity of $\nu=3$, and completely fails around $\nu=1$ where difference between σ^+ and σ^- peaks shows almost no enhancement effect. These discrepancies are due to the fact that the physics of PL processes for a 2DEG at low filling factors is far more complex[23–29] compared to our temptingly simple picture of electrons which recombine (are extracted) from the homogeneous Fermi sea of a 2DEG.

The use of discrete LLs in our calculations is justified by the large bare Zeeman energy which exceeds the LL width (110 μeV, extracted from low-field transport data) already at fields of $\sim$2 T ($\nu \sim 9$). The assumption that $\Delta E_{\uparrow\downarrow}$ does not depend on LL index is probably also realistic. When modeling the data, we have investigated various scenarios for a LL index dependence of the spin-gap enhancement but found that a constant value reproduces the data fairly well. Although it is more difficult to accurately determine the energy of the weak magneto-PL peaks for the higher $N>0$ LLs, it is possible to follow the separation between σ^+ and σ^- transitions associated with the L_1 level in the vicinity of $\nu=5$. As shown in Fig. 3 (right panel), the extracted enhancement of the spin gap in the L_1 level is practically the same as in the L_0 level. Moreover, we find a fair agreement between the spin gaps $|g|\mu_B B+\Delta E_{\uparrow\downarrow}$, extracted from PL, for fully populated LLs and the activation gaps, of 0.95, 0.63, and 0.36 meV, for spin excitations across the Fermi energy, which we have estimated from resistance measurements at filling factors $\nu=5$, 7, and 9, respectively.

Finally, let us speculate about a possible extension of the assumed model to the limit of low magnetic fields and to the particular case of $\nu=1$. When $B\rightarrow 0$, the extrapolation of Eq. (1) (at $T=0$ K) yields a linear Δ_S versus B dependence; $\Delta_S=|g|\mu_B B+\Delta_0/\nu=(|g|+\Delta_0/\mu_B B_{\nu=1})\mu_B B=g^*\mu_B B$, where $B_{\nu=1}$ corresponds to the magnetic field for $\nu=1$. With $B_{\nu=1}=18.5$ T ($n=4.5\times 10^{11}$ cm^{-2}) and $\Delta_0=2.1$ meV we extract $g^*=3.6$ for the enhanced g factor in good agreement with the estimation of $g^*\sim 3.7$ from the low field onset of spin splitting in the SdH oscillations. Setting $\nu=1$ (and $\mathcal{P}=1$) in Eq. (1) we extrapolate $\Delta_S=|g|\mu_B B+\Delta_0\sqrt[4]{1+B^2_{\nu=1}/B_0^2}$ and calculate $\Delta_S=6.4$ meV. This value is a factor of $\sim$4 smaller than its ultimate limit of $\sqrt{\pi/2}e^2/\epsilon l_B$ (Refs. 8 and 9) but in good agreement with the reported values in GaAs structures from optical and capacitance measurements.[10,14]

As shown in this paper, the magneto-PL in CdTe QWs reflects the simple picture of "many body enhanced" energy spacing between fully occupied 2D electron Landau levels. The identification of the energy difference between the σ^+ and σ^- PL lines with the difference of energies which are required to extract the spin-up and spin-down electron from occupied Landau levels is obviously a simplification. We think this simplification might be well justified in the limit of low magnetic fields, when $\beta=\frac{\hbar\omega_c}{e^2/\epsilon l_B}<1$ (in CdTe QWs: $\beta=1$ at $B=30$ T) and at high electron concentrations (high filling factors). On the other hand, for the dilute systems and at high magnetic fields, a more appropriate picture of the PL process may consist of considering the recombination between the interband magnetoexcitons in the initial state and spin or charge excitations of a 2DEG in the final sate.[23–29] Reasoning in terms of this latter approximation we also expect the polarization splitting of PL peaks to reflect only a bare Zeeman effect at even filling factors when spin polarization vanishes, but the analysis of σ^+ and σ^- PL to be more complex in the vicinity of low odd filling factors (e.g., $\nu=1,3$). This alternative approach might be more appropriate for GaAs-based structures, for which the high-field limit is reached at lower magnetic fields (in GaAs QWs $\beta=1$ at $B=8$ T).

VII. CONCLUSIONS

In conclusion, spectroscopic polarization-resolved magneto-PL studies of a 2DEG confined in CdTe quantum well reveal the many-body enhancement of the spin splitting of fully occupied 2D Landau levels well below the Fermi energy. The enhancement is mainly determined by the spin polarization of the 2DEG, since the spin gap is maximized at odd filling factors, but vanishes at even filling factors or high temperatures. We argue that the spin polarization may simply induce, in addition to the ordinary Zeeman splitting, a rigid shift of the spin-up Landau levels with respect to spin-down Landau levels. This simple picture for the many body spin-gap enhancement emerges from magneto-PL studies of a 2DEG with relatively large (single particle) g factor, in the limit of low magnetic fields ($\beta<1$).

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[1] R. J. Nicholas, R. J. Haug, K. v. Klitzing, and G. Weimann, Phys. Rev. B **37**, 1294 (1988).
[2] A. Usher, R. J. Nicholas, J. J. Harris, and C. T. Foxon, Phys. Rev. B **41**, 1129 (1990).
[3] D. R. Leadley, R. J. Nicholas, J. J. Harris, and C. T. Foxon, Phys. Rev. B **58**, 13036 (1998).
[4] D. K. Maude, S. Marty, L. B. Rigal, M. Potemski, J. C. Portal, Zb. Wasilewski, M. Henini, L. Eaves, G. Hill, and M. A. Pate, Physica B **249-251**, 1 (1998).
[5] F. F. Fang and P. J. Stiles, Phys. Rev. **174**, 823 (1968).
[6] J. F. Janak, Phys. Rev. **178**, 1416 (1969).
[7] T. Ando and Y. Uemura, J. Phys. Soc. Jpn. **37**, 1044 (1974).
[8] Yu. A. Bychkov, S. V. Iordanskii, and G. M. Éliashberg, Pis'ma Zh. Eksp. Teor. Fiz. **33**, 152 (1981) [JETP Lett. **33**, 143 (1981)].
[9] C. Kallin and B. I. Halperin, Phys. Rev. B **30**, 5655 (1984).
[10] V. T. Dolgopolov, A. A. Shashkin, A. V. Aristov, D. Schmerek, W. Hansen, J. P. Kotthaus, and M. Holland, Phys. Rev. Lett. **79**,

729 (1997).
[11] J. K. Wang, D. C. Tsui, M. Santos, and M. Shayegan, Phys. Rev. B **45**, 4384 (1992).
[12] S. A. J. Wiegers, M. Specht, L. P. Lévy, M. Y. Simmons, D. A. Ritchie, A. Cavanna, B. Etienne, G. Martinez, and P. Wyder, Phys. Rev. Lett. **79**, 3238 (1997).
[13] I. V. Kukushkin, N. J. Pulsford, K. von Klitzing, R. J. Haug, K. Ploog, H. Buhmann, M. Potemski, G. Martinez, and V. B. Timofeev, Europhys. Lett. **22**, 287 (1993).
[14] I. V. Kukushkin and V. B. Timofeev, Adv. Phys. **45**, 147 (1996).
[15] M. Potemski, Physica B **256-258**, 283 (1998).
[16] O. E. Dial, R. C. Ashoori, L. N. Pfeiffer, and K. W. West, Nature (London) **448**, 176 (2007).
[17] R. G. Clark, S. R. Haynes, A. M. Suckling, J. R. Mallett, P. A. Wright, J. J. Harris, and C. T. Foxon, Phys. Rev. Lett. **62**, 1536 (1989).
[18] S. L. Sondhi, A. Karlhede, S. A. Kivelson, and E. H. Rezayi, Phys. Rev. B **47**, 16419 (1993).
[19] A. Schmeller, J. P. Eisenstein, L. N. Pfeiffer, and K. W. West, Phys. Rev. Lett. **75**, 4290 (1995).
[20] D. K. Maude, M. Potemski, J. C. Portal, M. Henini, L. Eaves, G. Hill, and M. A. Pate, Phys. Rev. Lett. **77**, 4604 (1996).
[21] G. Karczewski, J. Jaroszyński, A. Barcz, M. Kutrowski, T. Wojtowicz, and J. Kossut, J. Cryst. Growth **184-185**, 814 (1998).
[22] B. A. Piot, J. Kunc, D. K. Maude, C. Betthausen, A. Vogl, D. Weiss, G. Karczewski, T. Wojtowicz, Phys. Rev. B **82**, 081307 (2010).
[23] K. Asano and T. Ando, Phys. Rev. B **65**, 115330 (2002), and references therein.
[24] N. R. Cooper and D. B. Chklovskii, Phys. Rev. B **55**, 2436 (1997).
[25] L. Gravier, M. Potemski, P. Hawrylak, and B. Etienne, Phys. Rev. Lett. **80**, 3344 (1998).
[26] P. Hawrylak and M. Potemski, Phys. Rev. B **56**, 12386 (1997).
[27] M. Byszewski, B. Chwalisz, D. K. Maude, M. L. Sadowski, M. Potemski, T. Saku, Y. Hirayama, S. Studenikin, D. G. Austing, A. S. Sachrajda, and P. Hawrylak, Nat. Phys. **2**, 239 (2006).
[28] O. Volkov, V. Zhitomirskii, I. Kukushkin, K. von Klitzing, and K. Eberl, JETP Lett. **65**, 38 (1997).
[29] F. Plentz, D. Heiman, L. N. Pfeiffer, and K. W. West, Phys. Rev. B **57**, 1370 (1998).
[30] R. Romestain and C. Weisbuch, Phys. Rev. Lett. **45**, 2067 (1980).
[31] Q. X. Zhao, M. Oestreich, and N. Magnea, Appl. Phys. Lett. **69**, 3704 (1996).
[32] Coulomb interaction scales with the inverse of the magnetic length at high fields (Refs. 7–9) but rather with the inverse of the mean interelectron distance when B tends to zero (Refs. 3, 6, and 7).